# Combinatorial Fourier transform for type A quiver representation varieties


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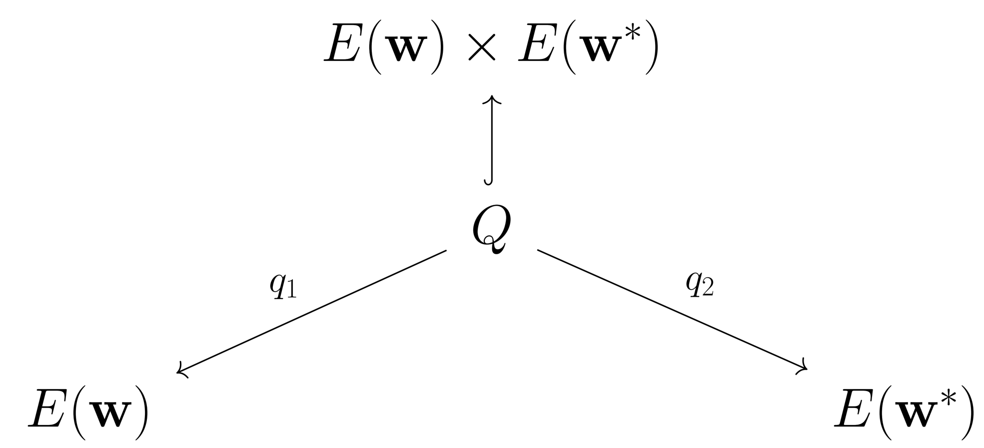
## Goal

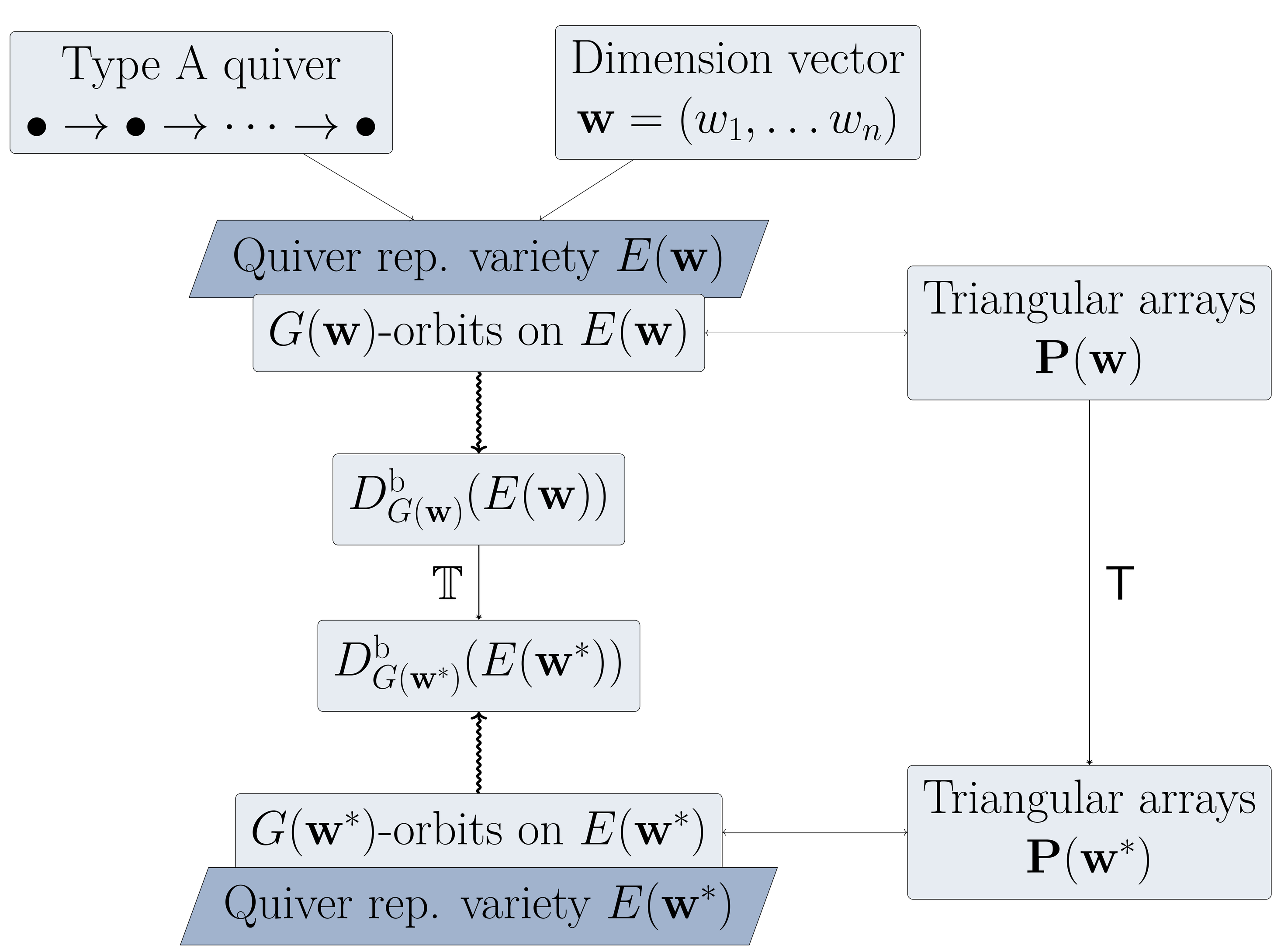
Give a combinatorial description of the Fourier–Sato transform:

$$D^{\mathrm{b}}_{G(\mathbf{w})}(E(\mathbf{w})) \xrightarrow{\mathbb{T}} D^{\mathrm{b}}_{G(\mathbf{w}^*)}(E(\mathbf{w}^*))$$
$$\mathcal{F} \longmapsto q_{2!}q_1^*(\mathcal{F})[\dim E(\mathbf{w})]$$

where



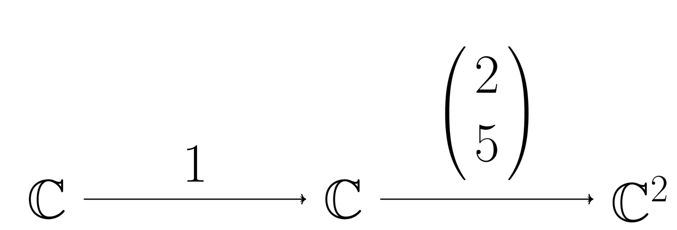

## Overview




## Quiver representations

A **quiver representation** is:

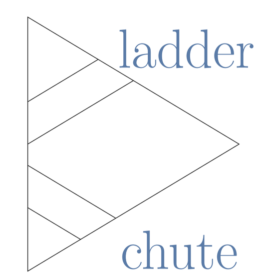
- A finite-dimensional $\mathbb{C}$-vector space for each vertex.
- A linear map for each arrow.




A **quiver representation variety** $E(\mathbf{w})$ is the space of all quiver representations for a fixed dimension vector $\mathbf{w}$.

$G(\mathbf{w}) = \mathbf{GL}(w_1) \times \ldots \times \mathbf{GL}(w_n)$ acts on $E(\mathbf{w})$ splitting it into orbits.

## The set of triangular arrays $\mathbf{P}(\mathbf{w})$




Define the set $\mathbf{P}(\mathbf{w})$ of triangular arrays of nonnegative integers such that:

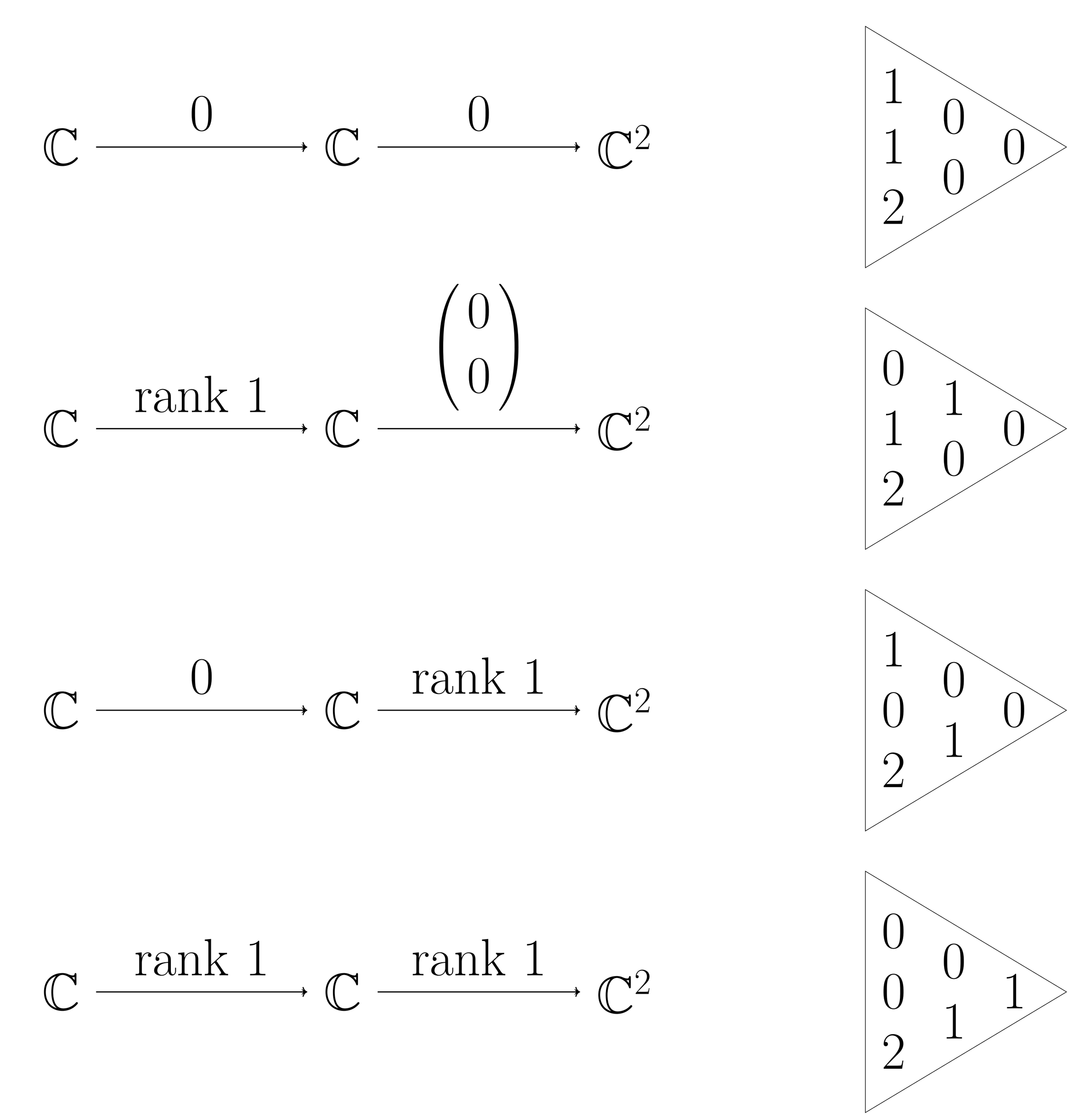
- $\forall j$, the entries in the $j^{\text{th}}$ chute sum to $w_j$.
- Ladders are weakly decreasing.

### Theorem [Achar–Kulkarni–M.]

There is a bijection

$$\{G(\mathbf{w})\text{-orbits in } E(\mathbf{w})\} \xleftrightarrow{1-1} \mathbf{P}(\mathbf{w}) = \{\text{certain tri. arrays}\}.$$

## Example of bijection

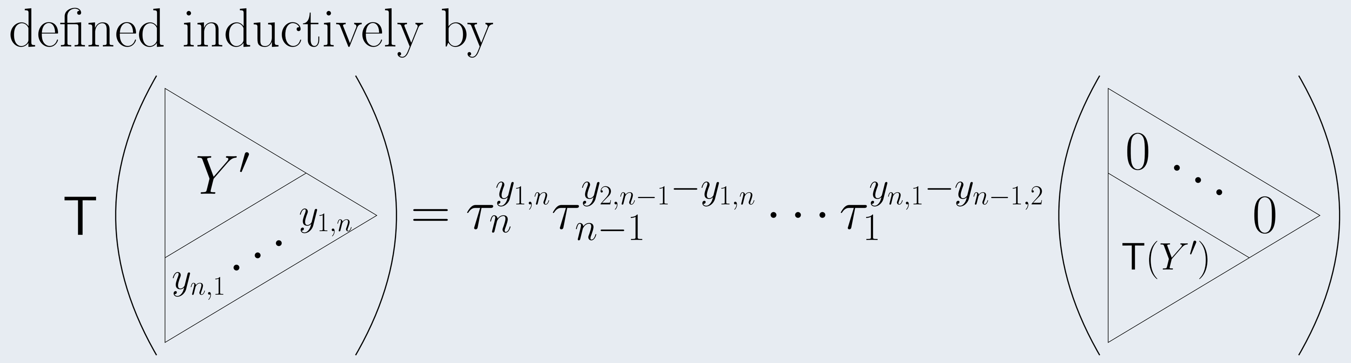



### Theorem (Combinatorial Fourier transform) [Achar–Kulkarni–M.]

There is a bijection

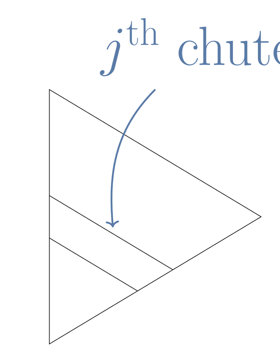
$$\mathbf{P}(\mathbf{w}) \xrightarrow{\mathsf{T}} \mathbf{P}(\mathbf{w}^*)$$

defined inductively by




where $\mathsf{T}(a) = a$.

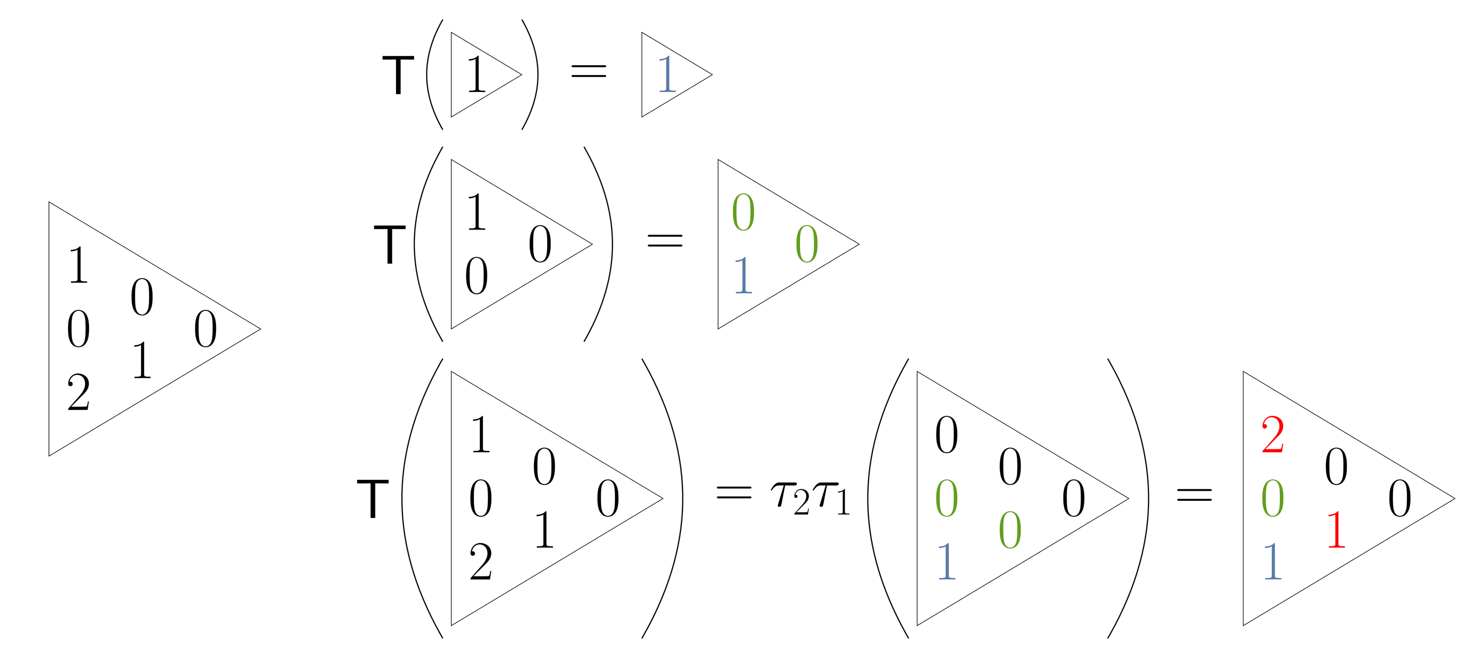
## Definition of $\tau_j$



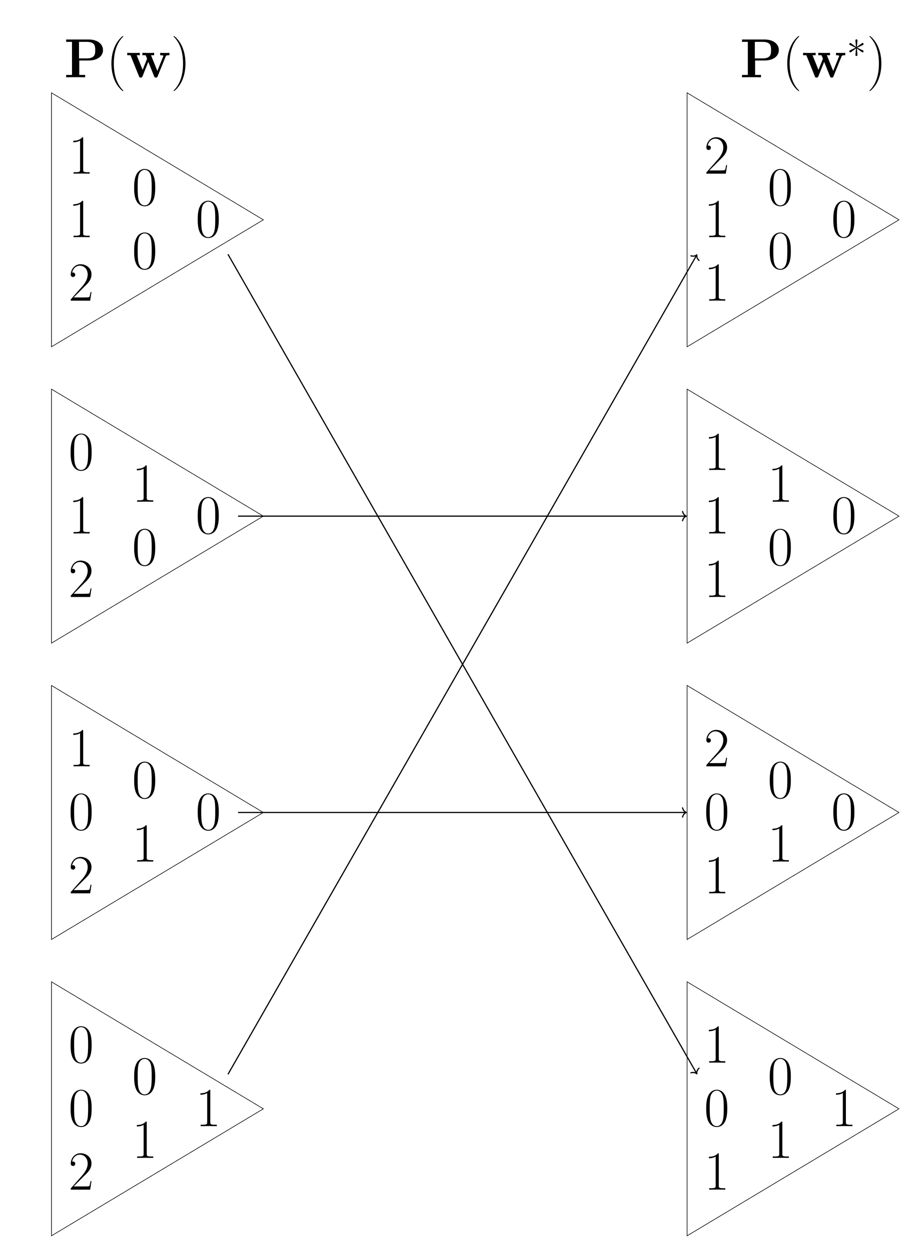

Define $\tau_j : \mathbf{P}(\mathbf{w}) \to \mathbf{P}(\mathbf{w} + \mathbf{e}_1 + \ldots + \mathbf{e}_j)$ by:

- Add 1 as far down the $j^{\text{th}}$ chute as possible, drawing an impassable vertical line there.
- Repeat for chutes $j-1, \ldots, 1$ not crossing lines.

## Example of CFT




## Complete CFT example




### Main conjecture (proof in progress)

The bijection $\mathsf{T}$ determines $\mathbb{T}$ on simple perverse sheaves; that is, $\mathbb{T}(\mathrm{IC}(\mathcal{O}_\lambda)) = \mathrm{IC}(\mathcal{O}_{\mathsf{T}(\lambda)})$.